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PAINLEVE TEST: DEFORMED COUPLED KDV TYPE NONLINEAR SCHR ODINGER EQUATION

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ABSTRACT

In this paper, we study an integrable system of deformed coupled Kdv type nonlinear Schrodinger equation (NLSE) and show that it passes Painleve test and therefore it is Painleve integrable. We also present complete group classi cation of this equation by obtaining point symmetries, corresponding reduced system of ordinary di erential equations (ODEs) and some particular solutions are also derived.

KEYWORDS: Point trasformations, Painleve test, deformed kdv type nonlinear Schrodinger equation.

INTRODUCTION

In the past few decades, nonlinear partial di erential equations (NLPDEs) are widely used to describe complex phenomena in various elds of sciences. Therefore solving NLPDE problems plays an important role in nonlinear sciences. In fact there are many di erent methods to obtain particular exact solution of NLPDEs. The most e ective methods are the Lie group method, the generalized conditional symmetry method, nonlocal symmetry method, etc.[3,4,12,13,17] We know that some of these equations are very di cult to solve, but much e ort has been made to construct the analytical solutions for them using any one of the above methods.

The integrability of NLPDEs is an interesting topic in nonlinear sciences. Many methods have been established by Mathematicians and Physicists to study the integrability of NLPDES. Some of the most important methods are the Painleve analysis method, Backland

and Darboux transformations, the symmetry reductions etc.[1,2,5,7,8,11,16,17] In this paper we report that the deformed coupled Kdv type NLSE.

$$u_t + u_{xxx} - 6\, uvux - g = 0 \qquad (1.1a)$$

$$v_t + v_{xxx} - 6\, uvvx - h = 0 \qquad (1.1b)$$

$$g_x + 2fu = 0 \qquad (1.1c)$$

$$h_x + 2fv = 0 \qquad (1.1d)$$

$$fx - uh + vg = 0 \qquad (1.1e)$$

Where u,v,g,h,f are functions of x,t with the Lax pair given in[8] passes the Painleve test for Partial di erential equations(PDEs) and therefore it is Painleve integrable.

The outline of the paper is as follows. In Section 2, we present Painleve test for the system of deformed coupled Kdv type NLSE (1.1) and show that it passes Painleve test and hence therefore it is Painleve integrable. In Section 3, we obtain point symmetries of deformed coupled Kdv type NLSE and then corresponding reduced system of ODEs and some particular solutions are derived. Finally, concluding remarks are summarized in Section 4.

Painleve Test

Painleve Test is a method of investigation for the integrability properties of many nonlinear evolution equations. If a PDE which has no movable branch, algebraic and logarithmic points then it is called Painleve integrable or P type. An ODE might still admit movable essential singularities without movable branch points. This method does not identify essential singularities and therefore it provides only necessary conditions for an ODE to be of Ptype. Singularity structure analysis admitting the Pproperty advocated by Ablowitz et al For ODEs and extended to PDE by Weiss, Tabor and Carnevale (WTC), plays a key role of investigating the integrability properties of many nonlinear evolution equations.[16,17] The wellknown procedure of WTC requires,

1. The determination of leading orders Laurent series.
2. The identi cation of powers at which the arbitrary functions can enter into the Laurent series called resonances.
3. Verifying that, at the resonance values, su cient number of arbitrary functions exist without introducing the movable critical manifold.

Now we use the Painleve Test for PDEs to equation (1.1).

(i) Determination of the leading order behaviour Let us assume that

$$u = u_0\chi^{\alpha_1}, \quad v = v_0\chi^{\alpha_2}, \quad g = g_0\chi^{\alpha_3}, \quad h = h_0\chi^{\alpha_4}, \quad f = f_0\chi^{\alpha_5}, \tag{2.1}$$

(ii) where $_1$; $_2$; $_3$; $_4$ and $_5$ are negative integers and u_0; v_0; g_0; h_0 and f_0 are functions of x,t and (x; t) is the singularity manifold. Substituting (2.1) in (1.1) and equating the most dominant terms we nd that

$$\alpha_1 = \alpha_2 = -1, \quad \alpha_3 = \alpha_4 = \alpha_5 = -2 \tag{2.2}$$

and then we substitute

$$u(x,t) = \frac{u_0}{\chi}, \quad v(x,t) = \frac{v_0}{\chi}, \quad g(x,t) = \frac{g_0}{\chi^2}, \quad h(x,t) = \frac{h_0}{\chi^2}, \quad f(x,t) = \frac{f_0}{\chi^2}, \tag{2.3}$$

into equation (1.1) and from the most dominant terms we nd that

$6\ u_0 v_0{}^2{}_x \quad 6\ v_{0x}{}^3 = 0$ (2.4b)

$$6\, u_0 v_0^2 \chi_x - 6\, v_0 \chi_x^3 = 0 \qquad (2.4b)$$

$$-u_0 h_0 + v_0 g_0 - 2 f_0 \chi_x = 0 \qquad (2.4c)$$

$$u_0 f_0 - g_0 \chi_x = 0 \qquad (2.4d)$$

$$v_0 f_0 + h_0 \chi_x = 0 \qquad (2.4e)$$

From equations (2.4a, b) we see that either u_0 or v_0 are arbitrary.

Without loss of generality, we can assume v_0 is arbitrary.

From equation (2.4a) we can find

$$u_0 = \frac{\chi_x^2}{v_0} \tag{2.5}$$

Using (2.5) in the remaining equations (2.4c, d) and (2.4e) we can find that

$$g_0 = \frac{f_0 \chi_x}{v_0}, \quad h_0 = -\frac{f_0 v_0}{\chi_x}$$

And f_0 is arbitrary.

Now we can nd the power at which the arbitrary functions enters into the series solution, For this purpose we subsititute the following into (1.1),

$$\begin{aligned} u(x,t) &= \frac{\chi_x^2}{v_0\chi} + u_r\chi^{r-1}, \\ v(x,t) &= \frac{v_0}{\chi} + v_r\chi^{r-1}, \\ g(x,t) &= \frac{f_0\chi_x}{v_0\chi^2} + g_r\chi^{r-2}, \\ h(x,t) &= -\frac{f_0v_0}{\chi_x\chi^2} + h_r\chi^{r-2}, \\ f(x,t) &= \frac{f_0}{\chi^2} + f_r\chi^{r-2} \end{aligned}$$

where u_r,v_r,g_r,h_r and f_r are functions of x,t and then equating the lowest order terms to zero we get the system of equations in u_r,v_r,g_r,h_r and f_r. In matrix form

$$A\,X = 0$$

$$\text{Where } A = \begin{pmatrix} (r^3-6r^2+5r+6)\chi_x^3 & \frac{6\chi_x^5}{v_0^2} & 0 & 0 & 0 \\ 6\chi_x v_0^2 & (r^3-6r^2+5r+6)\chi_x^3 & 0 & 0 & 0 \\ \frac{v_0 f_0}{\chi_x} & \frac{f_0\chi_x}{v_0} & (r-2)\chi_x & v_0 & \frac{-\chi_x^2}{v_0} \\ 2f_0 & 0 & \frac{2\chi_x^2}{v_0} & (r-2)\chi_x & 0 \\ 0 & -2f_0 & -2v_0 & 0 & (r-2)\chi_x \end{pmatrix}$$

and $X^T = (u_r, v_r, g_r, h_r, f_r)$.

$$\chi_x^9 r^2(r+1)(r-1)(r-2)(r-3)(r-4)(r-5)^2 = 0$$

For nding the resonance values at which the arbitrary function enters into series solution we take det (A) =0. This gives

Solving the above equation we get the resonance values,

R = 1,0,0,1,2, 3,4,4,5

Obviously, the resonance value at -1 represents the arbitrariness of (x; t). In order to compute arbitrary functions at resonance levels we substitute the following Laurents expansions into (1.1).

$$
\begin{aligned}
u(x,t) &= \frac{\chi_x^2}{v_0\chi} + \sum_{r=1}^{4} u_r\chi^{r-1}, \\
v(x,t) &= \frac{v_0}{\chi} + \sum_{r=1}^{4} v_r\chi^{r-1},
\end{aligned}
$$

$$
\begin{aligned}
g(x,t) &= \frac{f_0\chi_x}{v_0\chi^2} + \sum_{r=1}^{4} g_r\chi^{r-2}, \\
h(x,t) &= -\frac{f_0 v_0}{\chi_x\chi^2} + \sum_{r=1}^{4} h_r\chi^{r-2}, \\
f(x,t) &= \frac{f_0}{\chi^2} + \sum_{r=1}^{4} f_r\chi^{r-2}
\end{aligned}
$$

From the leading order behaviour analysis it is clear that v_0 and to the resonance levels 0 and 0. Equating the coe cients of $(\chi^{-3}, \chi^{-3}, \chi^{-2}, \chi^{-2}, \chi^{-2})$ in (1.1) to zero, we obtain the following linear system of equations in (u_1,v_1,g_1,h_1,f_1). Thay are

$$
\begin{aligned}
u_1\chi_x^3 + \frac{\chi_x^5 v_1}{v_0^2} &= -\frac{\chi_x^3\chi_{xx}}{v_0} \\
u_1 v_0^2\chi_x + \chi_x^3 v_1 &= -v_0\chi_x\chi_{xx} \\
\frac{v_1 f_0\chi_x}{v_0} + \frac{u_1 v_0 f_0}{\chi_x} - \frac{\chi_x^2 h_1}{v_0} + v_0 g_1 - f_1\chi_x &= -f_{0x} \\
-\frac{f_0\chi_x v_{0x}}{v_0^2} + \frac{f_{0x}\chi_x}{v_0} + \frac{f_0\chi_x^2}{v_0} + 2\,\frac{\chi_x^2 f_1}{v_0} - g_1\chi_x + 2\,u_1 f_0 &= 0 \\
\frac{v_0 f_0\chi_{xx}}{\chi_x^2} - \frac{v_{0x} f_0}{\chi_x} - \frac{v_0 f_{0x}}{\chi_x} - h_1\chi_x - 2\,v_0 f_1 - 2\,v_1 f_0 &= 0
\end{aligned}
$$

Solving the above system of linear equations we get g_1 is arbitrary at the resonance level 1. The explicit values of u_1; v_1; h_1 and f_1

$$
\begin{aligned}
u_1 &= \frac{\chi_x g_1 v_0^2 - 3\,v_0 f_0\chi_{xx} + f_0\chi_x v_{0x} + f_{0x}\chi_x v_0}{2 f_0 v_0^2} \\
v_1 &= \frac{-\chi_x g_1 v_0^2 + v_0 f_0\chi_{xx} - f_0\chi_x v_{0x} - f_{0x}\chi_x v_0}{2 f_0\chi_x^2} \\
f_1 &= \frac{f_0\chi_{xx} - f_{0x}\chi_x}{\chi_x^2} \\
h_1 &= -\frac{v_0\left(-\chi_x g_1 v_0 + 2\,f_0\chi_{xx} - 2\,f_{0x}\chi_x\right)}{\chi_x^3}
\end{aligned}
$$

Now equating the coe cients of $(\chi^{-2}, \chi^{-2}, \chi^{-1}, \chi^{-1}, \chi^{-1})$ in (1.1) to zero,we obtain the following linear system of equations in (u_2,v_2,g_2,h_2,f_2).

$$-\frac{f_0\chi_x}{v_0} - \frac{\chi_x^2\chi_t}{v_0} - 12\,\frac{\chi_{xx}^2\chi_x}{v_0} - 7\,\frac{\chi_x^2\chi_{xxx}}{v_0} + 3\,\frac{\chi_x^3 v_{0xx}}{v_0^2} - 6\,\frac{\chi_x^3 v_{0x}^2}{(v_0)^3} - 12\,u_1\chi_x\chi_{xx} + 6\,\frac{\chi_x^5 v_2}{v_0^2}$$

$$-6\,\chi_x^2 u_{1x} - 12\,\frac{\chi_x^3 v_1\chi_{xx}}{v_0^2} + 6\,\frac{\chi_x^4 v_1 v_{0x}}{v_0^3} + 15\,\frac{\chi_x^2\chi_{xx}v_{0x}}{v_0^2} + 6\,\frac{u_1 v_1\chi_x^3}{v_0} + 6\,\frac{u_1\chi_x^2 v_{0x}}{v_0} = 0$$

$$-6\,u_1 v_0 v_{0x} + 6\,u_2 v_0^2\chi_x - \frac{v_0 f_0}{\chi_x} - 3\,v_{0xx}\chi_x - v_0\chi_{xxx} - 3\,\chi_{xx}v_{0x} - v_0\chi_t$$

$$-6\,\chi_x^2 v_{1x} - 6\,\frac{\chi_x^2 v_1 v_{0x}}{v_0} + 6\,u_1 v_1 v_0\chi_x = 0$$

$$v_0 g_2 + v_1 g_1 - u_1 h_1 + \frac{u_2 v_0 f_0}{\chi_x} + \frac{v_2 f_0\chi_x}{v_0} + f_{1x} - \frac{\chi_x^2 h_2}{v_0} = 0$$

$$2\,u_1 f_1 + 2\,u_2 f_0 + g_{1x} + 2\,\frac{\chi_x^2 f_2}{v_0} = 0$$

$$-2\,v_0 f_2 - 2\,v_1 f_1 - 2\,v_2 f_0 + h_{1x} = 0$$

Solving the above system of equations with the values of (u_1,v_1,h_1,f_1) we nd that g_2 is arbitrary at the resonance level 2. The explicit values of (u_2,v_2,h_2,f_2) are obtained, due to lengthy ex-pressions we omit from presenting here. Similarly equating the coe cients of $(x^{-1}, x^{-1},x^0,x^0,x^0)$, (x^0,x^0,x^1,x^1,x^1) and then the coe cients of (x^1,x^1,x^2,x^2,x^2) in (2.1) to zero, we obatin the system of equations at each level, Solving the above systems we get h_3; f_4; h_4 and h_5 are arbitrary corresponding to the resonance levels 3,4,4 and 5. It is clear that equations (2.1) pass the Painleve test for PDEs and hence equation (2.1) is expected to be Painleve integrable.

Lie group Method

In the nineteenth century, Sophus Lie, introduced Lie groups to solve di erential equations,[12,13] when he discovered that the di erential equations are invariant under the continuous groups of transformations. The di erential equations can be reduced to simpler equations using its point symmetries.

Now let us consider a system of partial di erential equations as follows

$$\Delta_m(x, u^{(n)}) = 0, m = 1, 2, ..., l. \qquad (3.1)$$

where u = $(u_1, u_2, \ldots u_q)$, x = $(x_1, x_2, \ldots x_p)$; u^n denotes all the derivatives of u of all orders from 0 to n. The one-parameter Lie group of in nitesimal transformations of the system (3.1) is given by

$$x_i^* = x_i + \varepsilon\xi_i(x,u) + O(\varepsilon^2); i = 1, 2, ..., p,$$
$$u_{*\alpha} = u_\alpha + \varepsilon\eta_\alpha(x,u) + O(\varepsilon^2); \alpha = 1, 2, ..., q, \tag{3.2}$$

where " is the group parameter. The Lie algebra of (3.1) is spanned by vector field

$$X = \sum_{i=1}^{p} \xi_i(x,u)\frac{\partial}{\partial x_i} + \sum_{\alpha=1}^{q} \eta_\alpha(x,u)\frac{\partial}{\partial u_\alpha}$$

The n-th order prolongation of X is given by

$$X^n = \sum_{i=1}^{p} \xi_i(x,u)\frac{\partial}{\partial x_i} + \sum_{\alpha=1}^{q} \eta_\alpha(x,u)\frac{\partial}{\partial u_\alpha} + \sum_{\alpha=1}^{q}\sum_{J} \eta_\alpha^J(x,u^{(n)})\frac{\partial}{\partial u_\alpha^J}$$

Where J = $(i_1, \ldots i_k)$, $1 \leq i_k \leq p$, $1 \leq k \leq n$; and the sum is over all J s of order $0 < J \leq n$. If J = k, the coefficient η_α^J of $\frac{\partial}{\partial u_\alpha^J}$ depend only on k-th and lower order derivatives of u, and

$$\eta_\alpha^J(x,u^{(n)}) = D_J(\eta_\alpha - \sum_{i=1}^{p} \xi_i u_\alpha^i) + \sum_{i=1}^{p} \xi_i u_\alpha^{J,i},$$

$$\text{where } u_\alpha^i = \frac{\partial u_\alpha}{\partial x^i} \text{ and } u_\alpha^{J,i} = \frac{\partial u_\alpha^J}{\partial x^i}.$$

By considering the third prolongation of the above vector eld, under the constraints that the equations at hand be satis ed. The determining system lead to that the deformed coupled kdv type NLSE (1.1) is invariant under a one parameter (ϵ) continuous point transformations,

$$\begin{aligned} x^* &= x + \epsilon\xi_1 + O(\epsilon^2), \\ t^* &= t + \epsilon\xi_2 + O(\epsilon^2), \\ u^* &= u + \epsilon\eta_1 + O(\epsilon^2), \\ v^* &= v + \epsilon\eta_2 + O(\epsilon^2), \\ g^* &= g + \epsilon\eta_3 + O(\epsilon^2), \\ h^* &= h + \epsilon\eta_4 + O(\epsilon^2), \\ f^* &= f + \epsilon\eta_4 + O(\epsilon^2), \end{aligned}$$

where $\xi_1, \xi_2, \eta_1, \eta_2, \eta_3, \eta_4$ and η_5 are functions of x,t,u,v,g,h,f and v,h the conjugate of u,g respectively with the in nitesimal generator

$$X = \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial t} + \eta_1 \frac{\partial}{\partial u} + \eta_2 \frac{\partial}{\partial v} + \eta_3 \frac{\partial}{\partial g} + \eta_4 \frac{\partial}{\partial h} + \eta_5 \frac{\partial}{\partial f}$$

Where

$$\begin{aligned} \xi_1 &= c_1 x + c_2, \\ \xi_2 &= 3c_1 t + c_3 \\ \eta_1 &= (-c_1 + c_4)u, \\ \eta_2 &= -(c_1 + c_4)v \\ \eta_3 &= (-4c_1 + c_4)g, \\ \eta_4 &= -(4c_1 + c_4)h, \\ \eta_5 &= -4c_1 f \end{aligned} \qquad (3.3)$$

where c_1; c_2; c_3 and c_4 are constants,provided any solution of the dependent variables u,v,g,h,f satisfy the system (1.1).

For the above point transformations the in nitesimal generators are

$$\begin{aligned} X_1 &= \frac{\partial}{\partial t}, \\ X_2 &= \frac{\partial}{\partial x}, \\ X_3 &= u\frac{\partial}{\partial u} - v\frac{\partial}{\partial v} + g\frac{\partial}{\partial g} - h\frac{\partial}{\partial h} \\ X_4 &= 3t\frac{\partial}{\partial t} + x\frac{\partial}{\partial x} - u\,\frac{\partial}{\partial u} - v\,\frac{\partial}{\partial v} - 4\,g\,\frac{\partial}{\partial g} - 4\,g^*\,\frac{\partial}{\partial g^*} - 4\,f\frac{\partial}{\partial f}. \end{aligned}$$

and the Lie bracket table is given by

$$\begin{pmatrix} [X_j, X_k] & X_1 & X_2 & X_3 & X_4 \\ X_1 & 0 & 0 & 0 & 3X_1 \\ X_2 & 0 & 0 & 0 & X_2 \\ X_3 & 0 & 0 & 0 & \\ X_4 & -3X_1 & -X_2 & & 0 \end{pmatrix}$$

From the above table it is easy to see that the above in nitesimal generators forms a four dimensional Lie symmetry algebra.

The characteristic equation for the above in nitesimal generators is given by,

$$\begin{aligned}\frac{dx}{c_1x+c_2} &= \frac{dt}{3c_1t+c_3}=\frac{du}{(-c_1+c_4)u}=\frac{dv}{-(c_1+c_4)v}\\ &= \frac{dg}{(-4c_1+c_4)g}=\frac{dh}{-(4c_1+c_4)h}=\frac{df}{-4c_1f}\end{aligned} \tag{3.4}$$

If we choose the constant c_1 is zero in the above characteristic equation then we can nd the similarity variable and similarity transformations easily.

The similarity variable is
W (x,t) = x-k t

and similarity transformations are

$$\begin{aligned}u(x,t) &= U(w)\,e^{l\ t},\\ r(x,t) &= V(w)\,e^{-l\ t},\\ g(x,t) &= G(w)\,e^{l\ t},\\ h(x,t) &= H(w)\,e^{-l\ t},\\ f(x,t) &= F(w)\,.\end{aligned}$$

Where $k=\frac{c_3}{c_2}, l=\frac{c_4}{c_2}$ are constants.

We utilize the similarity variable and similarity transformations in system (1.1), we obtain the following reduced system of third order ODEs,

$$-U''' + k\,U' - l\,U + 6\,UVU' + G = 0 \tag{3.5a}$$
$$-V''' + k\,R' + l\,V + 6\,UVV' - H = 0 \tag{3.5b}$$
$$G' + 2\,U\,F = 0 \tag{3.5c}$$
$$H' - 2\,V\,F = 0 \tag{3.5d}$$
$$F' - U\,H + V\,G = 0 \tag{3.5e}$$

We employ the hyperbolic tangent method,[6,9,10,14,15] to obtain particular analytic solutions to the reduced system of ODES (3.5). For this purpose,
let us assume that y = tanhw.
Then the above reduced system (3.5) takes the following form

$$
\begin{aligned}
-(1-y^2)(-2y(-2yU-y+(1-y^2)U_{yy})-(1-y^2)(-2U_y-4yU_{yy}+(1-y^2)U_{yyy}))& \\
+k(1-y^2)U_y-l\,U+6\,UV(1-y^2)U_y+G &= 0 \quad (3.6a)\\
-(1-y^2)(-2y(-2yV-y+(1-y^2)V_{yy})-(1-y^2)(-2V_y-4yV_{yy}+(1-y^2)V_{yyy}))& \\
+k(1-y^2)V_y+l\,V+6\,UV(1-y^2)V_y-H &= 0 \quad (3.6b)\\
(1-y^2)G_y+2\,U\,F &= 0 \quad (3.6c)\\
(1-y^2)H_y-2\,V\,F &= 0 \quad (3.6d)\\
(1-y^2)F_y-U\,H+V\,G &= 0 \quad (3.6e)
\end{aligned}
$$

where U,V,G,H,F are functions of y and $U_y=\frac{dU}{dy}, U_{yy}=\frac{d^2U}{dy^2}, U_{yyy}=\frac{d^3U}{dy^3}$ and $V_y=\frac{dV}{dy}$,etc. On balancing the highest order derivative terms with the nonlinear terms one can easily nd that the solution will have the form as given by

$$
\begin{aligned}
U &= u_0+u_1y,\\
V &= v_0+v_1y\\
G &= g_0+g_1y+g_2y^2,\\
H &= h_0+h_1y+h_2y^2,\\
F &= f_0+f_1y+f_2y^2 \quad (3.7)
\end{aligned}
$$

Where $u_0,u_1,v_0,v_1,g_0,g_1,g_2$, etc are constants to be determined. Substituting the above expressions for U,V,G,H and F into (3.6), and performing the algebraic calculations on the relations obtained among various constants (that is,equating the coe cients of y equal to zero and solving them simultaneously we get all the values of u_0,u_1,v_0,etc).

For example in equation (3.6a), equate the coefficient of y^4 to zero, implies

$$-6u_1^2v_1+6u_1 = 0$$

From balancing the higher order derivative $u_1\neq 0, v_1\neq 0, g_2\neq 0, h_2\neq 0$ and $f_2\neq 0$ which implies

$$u_1 = \frac{1}{v_1}.$$

Similarly we can nd all the remaining constants. Equations (3.5) has the following particular solution,

$$\begin{aligned} U &= \frac{-1}{m\sqrt{6}} + \frac{\tanh w}{m}, \\ V &= \frac{m}{\sqrt{6}} + m\tanh w \\ G &= \frac{-2(k+1)}{3m} - \frac{(k+1)\sqrt{6}}{3m}\tanh w + \frac{(k+1)}{m}\tanh^2 w \\ H &= \frac{2m(k+1)}{3} - \frac{m(k+1)\sqrt{6}}{3}\tanh w - m(k+1)\tanh^2 w \\ F &= -(k+1) + (k+1)\tanh^2 w \end{aligned}$$

And Equations (1.1) has the following particular solution,

$$u(x,t) = \left(\frac{-1}{m\sqrt{6}} + \frac{\tanh(x-k\,t)}{m}\right)e^{l\,t}, \tag{3.8a}$$

$$v(x,t) = \left(\frac{m}{\sqrt{6}} + m\tanh(x-k\,t)\right)e^{-l\,t}, \tag{3.8b}$$

$$g(x,t) = \left(\frac{-2(k+1)}{3m} - \frac{(k+1)\sqrt{6}}{3m}\tanh(x-k\,t) + \frac{(k+1)}{m}\tanh^2(x-k\,t)\right)e^{l\,t}, \tag{3.8c}$$

$$h(x,t) = \left(\frac{2m(k+1)}{3} - \frac{m(k+1)\sqrt{6}}{3}\tanh(x-k\,t) - m(k+1)\tanh^2(x-k\,t)\right)e^{-l\,t}, \tag{3.8d}$$

$$f(x,t) = -(k+1) + (k+1)\tanh^2(x-k\,t), \tag{3.8e}$$

where $l = -\frac{(k+1)\sqrt{6}}{3}$.

If $k = -1$ in (3.6) then

$$\begin{aligned} u(x,t) &= \frac{-1}{m\sqrt{6}} + \frac{\tanh(x+t)}{m}, \\ v(x,t) &= \frac{m}{\sqrt{6}} + m\tanh(x+t), \end{aligned}$$

Is the solution of the coupled Kdv type NLSE.

Now we look for the rational type solution of (1.1).

Let us assume that $y = \frac{1}{w}$.

Then the above reduction equations (3.5) takes the following form

$$-(-y^2)(-2y(-2yU - y + (-y^2)U_{yy}) - (-y^2)(-2U_y - 4yU_{yy} + (-y^2)U_{yyy})) + k(-y^2)U_y - l\,U + 6\,UV(-y^2)U_y + G = 0 \tag{3.9a}$$

$$-(-y^2)(-2y(-2yV - y + (-y^2)V_{yy}) - (-y^2)(-2V_y - 4yV_{yy} + (-y^2)V_{yyy})) + k(-y^2)V_y + l\,V + 6\,UV(-y^2)V_y - H = 0 \tag{3.9b}$$

$$(-y^2)G_y + 2\,U\,F = 0 \tag{3.9c}$$

$$(-y^2)H_y - 2\,V\,F = 0 \tag{3.9d}$$

$$(-y^2)F_y - U\,H + V\,G = 0 \tag{3.9e}$$

Substituting (3.7) into (3.9) we arrive a set of algebraic equations by equating the coe cients of y equal to zero and solving them simultaneously we get all the values of u_0; u_1; v_0,etc. In equation (3.9a) the coe cient of y^4 is equal to zero, implies,

$$-6u_1^2 v_1 + 6u_1 = 0$$

Since u1 $\neq$ 0 which implies $u_1 = \frac{1}{v_1}$

Equations (3.5) has the following particular solution,

$$\begin{aligned} U &= \frac{-1}{m\sqrt{6}} + \frac{1}{m\ w}, \\ V &= \frac{m}{\sqrt{6}} + \frac{m}{w} \end{aligned}$$

$$\begin{aligned} G &= \frac{(k-1)}{3m} - \frac{(k-1)\sqrt{6}}{3m\ w} + \frac{(k-1)}{mw^2} \\ H &= -\frac{m(k-1)}{3} - \frac{m(k-1)\sqrt{6}}{3w} - \frac{m(k-1)}{w^2} \\ F &= \frac{(k-1)}{w^2} \end{aligned}$$

And Equations (1.1) has the following particular solution,

$$\begin{aligned} u(x,t) &= \left(\frac{-1}{m\sqrt{6}} + \frac{1}{m\ (x-k\,t)}\right) e^{l\ t}, \\ v(x,t) &= \left(\frac{m}{\sqrt{6}} + \frac{m}{(x-k\,t)}\right) e^{-l\ t}, \\ g(x,t) &= \left(\frac{(k-1)}{3m} - \frac{(k-1)\sqrt{6}}{3m\ (x-k\,t)} + \frac{(k-1)}{m(x-k\,t)^2}\right) e^{l\ t}, \\ h(x,t) &= \left(-\frac{m(k-1)}{3} - \frac{m(k-1)\sqrt{6}}{3(x-k\,t)} - \frac{m(k-1)}{(x-k\,t)^2}\right) e^{-l\ t}, \\ f(x,t) &= \frac{(k-1)}{(x-k\,t)^2}, \end{aligned}$$

Is the solution of the coupled Kdv type NLSE

where $l = -\frac{(k-1)\sqrt{6}}{3}$.
If $k = 1$, then

$$u(x,t) = \frac{-1}{m\sqrt{6}} + \frac{1}{m\,(x-t)},$$
$$v(x,t) = \frac{m}{\sqrt{6}} + \frac{m}{(x-t)}$$

CONCLUSION

In this paper, we study an integrable system of deformed coupled Kdv type nonlinear Schrodinger equation (NLSE) and shown that it passes pain eve test and therefore it is Pain eve inferable. We also present complete group classication of this equation by obtaining point symmetries, corresponding reduced system of ODEs and some particular solutions are also derived.

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