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COMPARATIVE STUDY OF THE MOTION OF TWO CONNECTED SATELLITES

***Dr. Sushil Chandra Karna**

Department of Mathematics M.B.M. Campus Rajbiraj, Saptari (Nepal).

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***Corresponding Author**
Dr. Sushil Chandra Karna
Department of Mathematics
M.B.M. Campus Rajbiraj,
Saptari (Nepal).

ABSTRACT

This paper is concerned with the motion of two satellites connected by light flexible, inextensible and non-conducting string in the Earth's oblatiness and Earth's magnetic field. We disused the comparative study of the motion on of connected two satellites.

KEYWORDS: Magnetic filed, Obtatiness, light flexible, inextensible.

INTRODUCTION

The linearised and normalized equations of relative motion of two cable connected satellites system in two cable connected satellites system in the filed of force of the oblate gravitating earth together with magnetic force.

We come to know that the totality of motion will consists of two phase as free and constraind motion. The satellites have been considered as material particles moving in the gravitational filed of oblate Earth and in Lorentz force filed (magnetic force). The cable connecting the satellites is assumed to be perfectly light, flexible and inextensible. The central part of the Earth potential will be taken as main force and other forces such as the remaining proton of the earth potential are mere disturbing forces of perturbative native. The centre of mass of the system of two cable – connected satellites in the central gravitational field of attraction of the earth moves along a Keplerian orbit. The impact suffered by the satellites at the time of tightening of the string is perfectly elastic in natural. The centre of mass was assured to move along a Keplerian elliptical orbit which mass in particular (upto second order of infinite).

As it is well known, the equation becomes non-linear due to the pressecuve of term containing earth's magnetic field $\vec{H}$

$$\vec{H} = -\nabla\left(\frac{\vec{M}.\vec{r_i}}{r_i^3}\right)$$

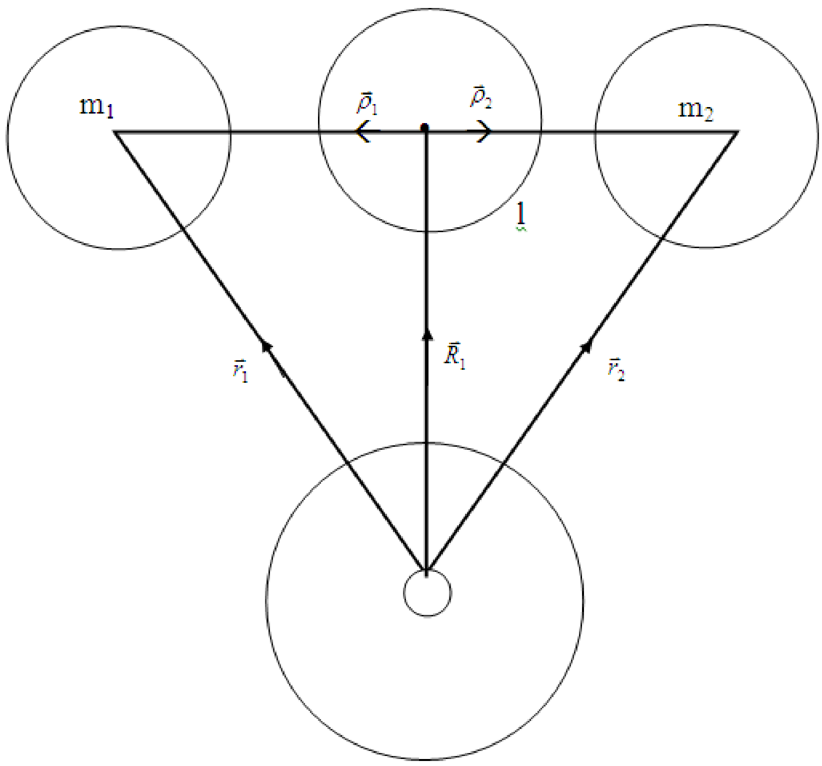
Where $i = 1,2$

$\nabla =$ Laplacial Operator

$\vec{M} =$ Magnetic moment of the earth

The length of the string connecting the two particles is infinitesimally small in compression to the distance of the centre of mass of the system from the centre of attracting force and that the innovation of the rotating from of reference eliminates product terms as usual.

Equation of the motion of the Center of Mass



Suppose m_1 & m_2 are two particles. Let $\vec{r_1}$ & $\vec{r_2}$ be the radians vectors of the particle m_1 & m_2 with respect to the gravitating centre O. $\vec{R}$ be the length of the light, flexible and inextensible string connecting them.

One condition for the constraint of the system is given by $\left|\vec{r_1} - \vec{r_2}\right| \le l$ one above expression explain that the distance between the two satellites can not exceed the length of the string. By using Lagrang's equation of the motion of first kind (Vector equation) as for the particle $m_1 \& m_2$

$$m_1 \ddot{\vec{r}}_1 = -\frac{\mu m_1 \vec{r}_1}{r_1^3} + \lambda\left(\vec{r}_1 - \vec{r}_2\right) - \frac{3m_1 \mu k_2 \vec{r}_1}{r_1^5} + Q_1\left(\vec{r}_1 \times \vec{H}\right)$$

$$m_2 \ddot{\vec{r}}_2 = -\frac{\mu m_2 \vec{r}_2}{r_2^3} + \lambda\left(\vec{r}_2 - \vec{r}_1\right) - \frac{3m_2 \mu k_2 \vec{r}_2}{r_2^5} + Q_2\left(\vec{r}_2 \times \vec{H}\right)$$

Where

$$\vee = \frac{\mu}{R} + \bar{e}\frac{R^2 e\mu}{3R^3}$$

R = Radians vector of the satellite from attracting centre

$$\bar{e} = \alpha R - \frac{m}{2}$$

$$m = \frac{\Omega^2 \text{ Re}}{ge}$$

Re = Equatorial radians of the earth

$\Omega =$ Angnbar velocity of the rotation of the earth.

$\alpha R =$ oblateness of the earth = $\frac{\text{Re} - Rp}{\text{Re}}$

$Rp =$ Polar radians of the earth

$$k_2 = \frac{\bar{e}R^2 e}{3}$$

$\lambda =$ Lagrange's undermined multiplier

$Q_i =$ charge q_i on the ith particle / velocity of light (C)

$\mu =$ product of gravitational constant and the mass of the earth

$\vec{H} =$ The intensity of earth's magnetic field for the equatorial satellite

$$= -\nabla\left(\frac{\vec{M}.\vec{r}}{r_3}\right)$$

M = Magnetic moments of earth

Let $\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

$\vec{R} =$ radians vector of the centre of mass

Adding

$$M\ddot{\vec{R}} = -\mu\left[\frac{m_1\vec{r_1}}{r_1^3}+\frac{m_2\vec{r_2}}{r_2^3}\right]-3\mu k_2\left[\frac{m_1\vec{r_1}}{r_1^5}+\frac{m_2\vec{r_2}}{r_2^5}\right]+\left[Q_1\dot{\vec{r_1}}+Q_2\dot{\vec{r_2}}\right]\times\vec{H}$$

Where

M = m1 + m2

Let

$\vec{\rho}_i$ = Radians vector of the particle

$\vec{r_1} = \vec{R}+\vec{\rho}_i$ (i = 1,2)

The relative motion of the system with respect to the centre of mass

$$m_1\vec{\rho_1}+m_2\vec{\rho_2}=0$$

The centre of mass is the origin of the reference frame. We observe that the maximum distance between the particle is much smaller in compression to the radians vector of their centre of mass with origin at the attracting center.

$\vec{\rho_i} < l < r_i$ (i = 1,2)

$r_i \sim R$

Again,

$$M\ddot{\vec{R}} = -\mu\left[\frac{m_1(\vec{R}+\vec{\rho_1})}{(\vec{R}+\vec{\rho_1})^3}+\frac{m_2(\vec{R}+\vec{\rho_2})}{(\vec{R}+\vec{\rho_2})^3}\right]-3\mu k_2\left[\frac{m_1(\vec{R}+\vec{\rho_1})}{(\vec{R}+\vec{\rho_1})^5}+\frac{m_2(\vec{R}+\vec{\rho_2})}{(\vec{R}+\vec{\rho_2})^5}\right]+\left[Q_1(\dot{\vec{R}}+\dot{\vec{\rho_1}})+Q_2(\dot{\vec{R}}+\dot{\vec{\rho_2}})\right]\times\vec{H}$$

Here

$$\frac{1}{(\vec{R}+\vec{\rho_2})^3} = \frac{1}{\left[(\vec{R}+\vec{\rho_2})^2\right]\frac{3}{2}}$$

$$= \frac{1}{\left[R^2+3\vec{R}.\vec{\rho}+\vec{\rho_1}^2\right]\frac{3}{2}}$$

$$= \frac{1}{R^3}\left[1+2\frac{\vec{R}.\vec{\rho_1}}{R^2}\right]\frac{-3}{2}$$

$$\frac{1}{(\vec{R}+\vec{\rho_1})^3} = \frac{1}{R^3}-3\frac{\vec{R}.\rho_1}{R^5}$$

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$$\frac{1}{\left(\vec{R}+\vec{\rho_2}\right)^3}=\frac{1}{R^3}-\frac{3\vec{R}.\vec{\rho_2}}{R^7}$$

$$\frac{1}{\left(\vec{R}+\vec{\rho_1}\right)^5}=\frac{1}{R^5}-\frac{5\vec{R}.\vec{\rho_1}}{R^7}$$

$$\frac{1}{\left(\vec{R}+\vec{\rho_2}\right)^5}=\frac{1}{R^5}-\frac{5\vec{R}.\vec{\rho_2}}{R^7}$$

With the help of $\vec{r_i}=\vec{R}+\vec{\rho_i}$

$$m_1\vec{\rho_1}+m_2\vec{\rho_2}=0$$

Then,

$$\left[Q_1\left(\dot{\vec{R}}+\dot{\vec{\rho_1}}\right)+Q_2\left(\dot{\vec{R}}+\dot{\vec{\rho_2}}\right)\right]\times\vec{H} \qquad =\left(Q_1+Q_2\right)\left[(\dot{\vec{R}}\times\vec{H})+(Q_1\dot{\vec{\rho_1}}+Q_2\dot{\vec{\rho_2}})\times\vec{H}\right]$$

$$\left(Q_1+Q_2\right)\left(\vec{R}+\vec{H}\right)$$

Again

$$M\ddot{\vec{R}}=\frac{-\mu\left(m_1+m_2\right)}{R^3}\vec{R}-\frac{\mu}{R^3}\left(m_1\vec{\rho_1}+m_2\vec{\rho_2}\right)+\frac{3\mu\vec{R}}{R^5}\left[(\vec{R}.\vec{\rho_1})m_1+\left(\vec{R}.\vec{\rho_2}\right)m_2\right]+\frac{3\mu}{R^5}\left[m_1\vec{\rho_1}\left(\vec{R}.\vec{\rho_1}\right)+m_2\vec{\rho_2}\left(\vec{R}.\vec{\rho_2}\right)\right]$$

$$-3\mu K_2\frac{\left(m_1+m_2\right)\vec{R}}{R^5}-\frac{3\mu K_2}{R^5}\left(m_1\vec{\rho_1}+m_2\vec{\rho_2}\right)$$

$$+\frac{15\mu K_2\vec{R}}{R^7}\left[m_1\left(\vec{R}.\vec{\rho_1}\right)+m_2\left(\vec{R}.\vec{\rho_2}\right)\right]$$

$$+\frac{15\mu K_2\vec{R}}{R^7}\left[m_1\vec{\rho_1}\left(\vec{R}.\vec{\rho_1}\right)+m_2\vec{\rho_2}\left(\vec{R}.\vec{\rho_2}\right)\right]+\left(Q_1+Q_2\right)\left(\dot{\vec{R}}\times\vec{H}\right)$$

Here

$$\vec{\rho_1}=\frac{m_2}{m_1+m_2}\left(\vec{r_1}-\vec{r_2}\right) \qquad \vec{\rho_2}=\frac{m_1}{m_1+m_2}\left(\vec{r_2}-\vec{r_1}\right)$$

$$\therefore m_1\vec{\rho_1}+m_2\vec{\rho_2}=0$$

After using the above expression in

$$M\ddot{\vec{R}}=\frac{-\mu M\vec{R}}{R^3}-\frac{3\mu K_2 M\vec{R}}{R^5}+\frac{3\mu\vec{R}}{R^5}\left[m_1(\vec{R}.\vec{\rho_1})+m_2\left(\vec{R}.\vec{\rho_2}\right)\right]+\frac{3\mu}{R^5}\left[m_1\vec{\rho_1}\left(\vec{R}.\vec{\rho_1}\right)+m_2\vec{\rho_2}\left(\vec{R}.\vec{\rho_2}\right)\right]$$

$$+\frac{15\mu K_2 \vec{R}}{R^7}\left[m_1\vec{\rho_1}\left(\vec{R}.\vec{\rho_1}\right)+m_2\vec{\rho_2}\left(\vec{R}.\vec{\rho_2}\right)\right]+\left(Q_1+Q_2\right)\left(\vec{R}\times\vec{H}\right)$$

We conclude that $\frac{1}{R^5}\,\&\,\frac{1}{\vec{R}}$

Becomes very small and may be neglected.

We have:

$$M\ddot{\vec{R}}+\frac{\mu M\vec{R}}{R^3}-\left(Q_1+Q_2\right)\left(\vec{R}\times\vec{H}\right)=0$$

The equation of energy be

$$\vee^2=\overrightarrow{\dot{R}^2}=\mathrm{A}+\frac{2\mu}{R}$$

A = constant

In this way, we observe that the centre of mass may be assumed to move along a Keplerial elliptical orbit in particular case may be circulars.

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